Merging of non–symmetric Burgers vortices

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The Burgers vortex is a well–known equilibrium solution to the Navier–Stokes equations in which viscous diffusion is balanced by vorticity intensification due to the stretching strain field. The Burgers vortex has been used to model various features of turbulence, and recent numerical simulations of turbulence have renewed interest in the properties of Burgers vortices.

Robinson & Saffman [4] calculated numerically steady solutions for a single vortex in a non-symmetric axial strain field for small Reynolds numbers Re. They showed how Re and the strain ratio λ (see below for definition) affected the ellipticity and orientation of the vortex. Buntine & Pullin [1] computed the time dependent merger of two vortices in an axisymmetric strain field for a range of Reynolds numbers $0.1 < Re < 1280$. They showed that the interaction of the vortices produced spiral structures and eventual relaxation to an axisymmetric Burgers vortex.

Moffatt, Kida & Ohkitani [2] developed a large Reynolds number asymptotic theory of stretched vortices in a non-symmetric straining field. Prochazka & Pullin [3] extended on the work of [2] and [4], and developed quasi–steady non–symmetric solutions for high Reynolds number and large biaxial strain. They also developed an asymptotic form for the vorticity enclosed within a *cat's–eye* region of nearly two-dimensional flow.

In this paper, we present the numerical study of the interaction between Burgers vortices in a non-symmetric background straining field, $(\alpha x, \beta y, \gamma z)$, where $\alpha + \beta + \gamma = 0$. We consider cases in which one principal extensional strain is aligned with the vorticity, and the value of the strain ratio, $\lambda = (\alpha - \beta)/(\alpha + \beta) > 0$.

The hybrid spectral finite–difference method of Buntine & Pullin [1] is used to solve the Navier–Stokes equations on an infinite domain. A vortex Reynolds number $Re = \Gamma/2\pi\nu$ is defined, where Γ is the total circulation and ν the kinematic viscosity. All quantities are non-dimensionalised using a length scale of $\delta = (2\nu/\gamma)^{1/2}$ and a time scale of $\tau = 2/\gamma$.

Initially, two Burgers vortices were placed symmetrically about the origin, separated by a distance of r_0. Figure 1 shows a representative simulation for which $Re = 10000, \lambda = 150$ and $r_0 = 5$. Although the strain field is extremely biaxial, the vortices rotate about each other, while the distance between their cores oscillates. Perhaps the most striking feature of the vorticity contour plots is the development of 'inner' and 'outer' spiral arm structures. The vorticity in the spiral arms becomes more concentrated and the 'outer' spiral arms develop into two 'tails' of vorticity that are convected away from the vortex.

We also plot contours of the rate of viscous dissipation due to the vortices, $D(r,\theta)$ for the merging events (Moffatt, Kida & Ohkitani [2]). Initially, the regions of high rate of dissipation do not overlap with the regions of high enstrophy. However, at later times, the dissipation becomes concentrated in the regions of high vorticity gradient inside the spiral arms.

Many questions remain unanswered about the behaviour of stretched vortices embedded in biaxial strain fields. Areas for future investigation include: the conditions on the initial core separation under which the vortices merge; the effect of the merging process on the relaxation towards the asymptotic and quasi–steady solutions of Prochazka & Pullin [3]; and the effect of more general velocity perturbations on the merging event, such as a time dependent strain.

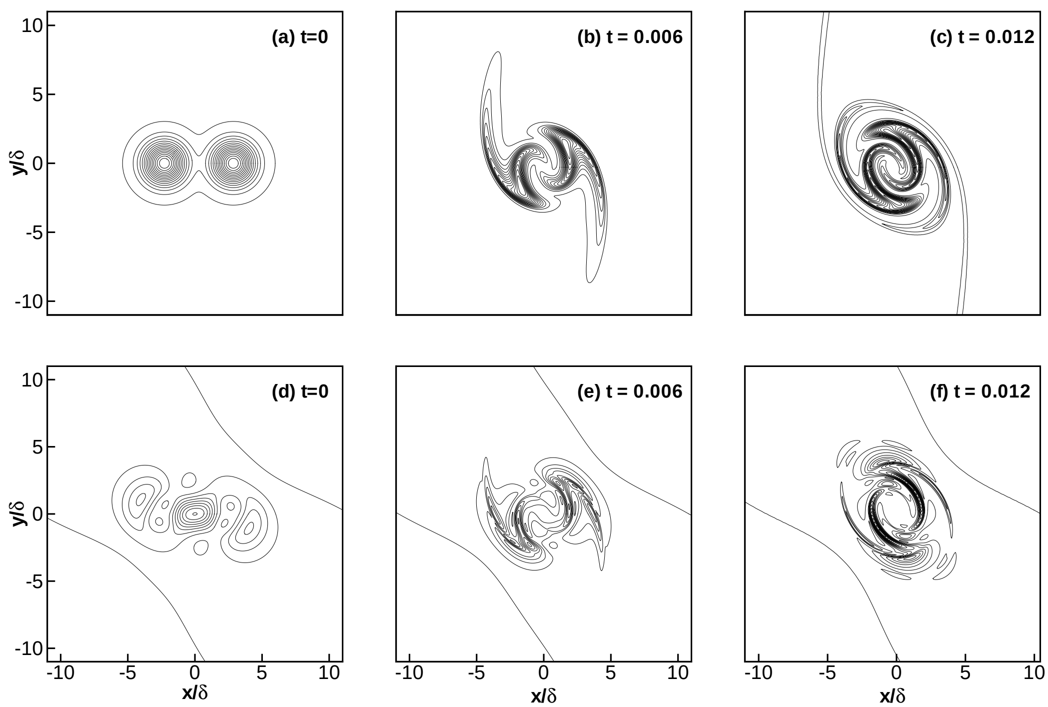

Figure 1: (a)–(c) Contours of vorticity, (d)–(f) contours of rate of viscous dissipation, $D(r,\theta)$. $Re = 10000, \lambda = 150$.

References

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