Wavelet approach to study tubes, sheets and singularities in turbulent flows

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The wavelet representation [1] uses self-similar localized functions and is therefore better suited than Fourier or windowed Fourier representations to study coherent structures and detect quasi-singularities in fully-developed turbulent flows.

In [2] it has been shown, using the continuous wavelet transform and wavelet based diagnostics as the intermittency measure and the space-scale Reynolds number, that the coherent structures observed in DNS (Direct Numerical Simulation) of 3D mixing layer and 3D channel flow are multiscale and responsible of the flow intermittency. They are characterized by their space-scale correlation and by a Reynolds number larger than one, all along the inertial range. The dissipative regions are also multiscale, but non intermittent. They lack space-scale correlation and have a Reynolds number close to one.

In [3, 4] a wavelet-based method extracting coherent structures out of turbulent flows has been applied to 3D homogeneous turbulent flow computed by DNS at resolution $N = 256^3$, which corresponds to a microscale Reynolds number $R_\lambda = 168$. Each flow realization is split into two orthogonal components, the coherent flow and the incoherent flow. The coherent vorticity and velocity fields are reconstructed from the $3\%N$ wavelet coefficients containing 99% of the energy, 79% of the enstrophy and retaining the coherent vortex tubes (Fig. 1b). The incoherent vorticity and velocity fields are reconstructed from the remaining $97\%N$ wavelet coefficients and are structureless (Fig. 1c). Both components are multiscale, but have different statistical behaviors (Fig. 2a). The coherent flow has long-range correlation, with the same $k^{-5/3}$ energy spectrum as the total flow. In contrast, the incoherent flow is decorrelated, with an equipartition energy spectrum and a Gaussian velocity PDF (Fig. 2b).

The PDF of the relative helicity (Figs. 2c) shows that the coherent flow has the same tendency towards Beltramization as the total flow, with two maxima corresponding to alignment and anti-alignment between the velocity and vorticity vectors. In contrast, the incoherent flow has a completely different behavior, with a tendency towards zero helicity which corresponds to two-dimensionalization. These observations support Moffatt's conjecture [5] stating that: *'Blobs of maximal helicity may be interpreted as coherent structures, separated by regular surfaces on which vortex sheets, the site of strong dissipation, may be located'*.

Since we have shown that the coherent flow contains the vortex tubes and has the same $k^{-5/3}$ power-law behavior as the total flow, we propose the following picture to explain the turbulent cascade. The transfer of energy between the various scales does not result from vortex fragmentation, as in the classical Richardson's scenario. It is instead due to nonlinear vortex interactions, *e.g.* stretching and straining, which transfer coherent energy throughout the whole inertial range, while at the same time producing incoherent energy which is dissipated at the smallest scales.

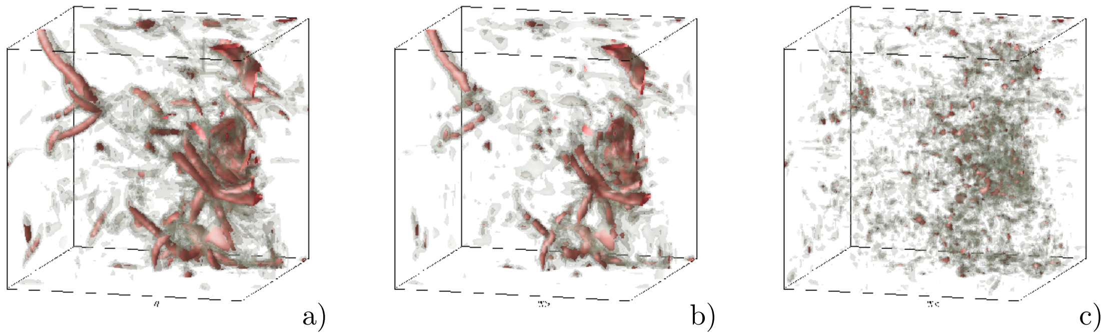

Figure 1: Homogeneous isotropic turbulence. Isosurfaces of vorticity modulus for a) total flow, b) coherent part, c) incoherent part.

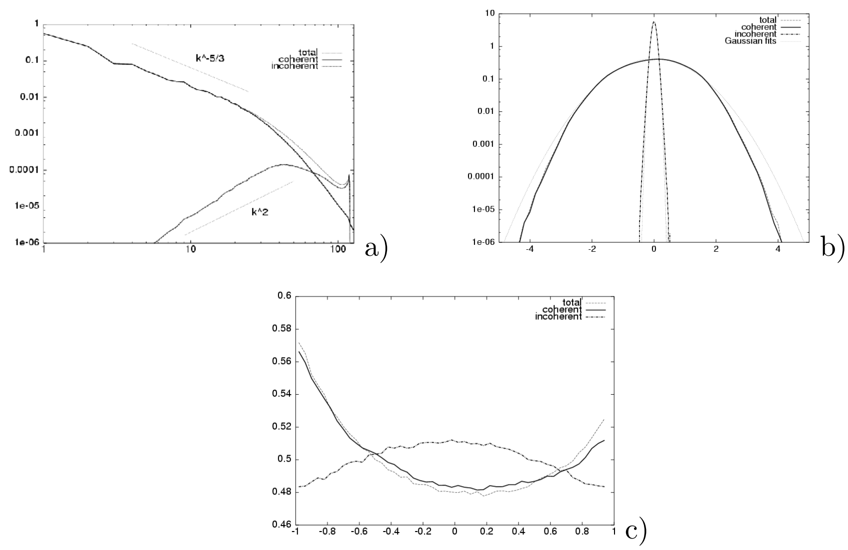

Figure 2: Superposition of the total, coherent and incoherent contributions to a) PDF of velocity b) energy spectra and c) PDF of relative helicity.

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References

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