An exponentially small massacre of BLT and DNS

Stephen Cowley

DAMTP, University of Cambridge
S.J.Cowley@damtp.cam.ac.uk

The classical unsteady boundary-layer equation is a well-known asymptotic approximation of the Navier-Stokes equations that admits solutions with finite-time singularities. These 'separation' singularities are believed to be an artifact of the boundary-layer approximation, and are *not* thought to indicate the development of a singularity in the Navier-Stokes equations.

Until recently, a separation singularity was thought to be the only [mathematical] phenomena by which the classical unsteady boundary-layer equations might cease to be an asymptotic approximation to the Navier-Stokes equations. However, Brinckman & Walker [1] have shown numerically that, even in the absence of noise and rounding error, asymptotically short wavelength, rapidly growing, Rayleigh instabilities can apparently be excited in certain circumstances. Such instabilities can lead to small-scale structures developing in the boundary-layer flow before the development of the separation singularity. We provide a [heuristic] asymptotic justification of these numerical results. In support of our argument we study a model problem governed by the Kuramoto-Sivashinsky equation and show, analytically, that a similar phenomena can occur in that case.

We argue that the possibility of exciting such rapidly growing short-wavelenth instabilities by large-scale mode-mode interactions, rather than by noise or rounding error, has distressing consequences for boundary-layer theory, and places a severe constraint on the Reynolds number for which accurate DNS solutions can be found using mainstream computers if, say, there is a rigid surface. Indeed it can be argued that the precision of calculations can be as severe a constraint as raw computational power in obtaining accurate solutions. The relevance of these results to numerical solutions of the Euler equations might be speculated on!

References

[1] Brinkman, K.W. and Walker, J.D.A. (2001). Instability in a viscous flow driven by streamwise vortices. *J. Fluid Mech.*.