Near Identity Transformations for the Navier-Stokes equations

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The mathematical theory of the Navier-Stokes equations is incomplete and requires cut-offs. The present state of knowledge is such that different approximations seem to be useful for different purposes. In this talk I describe some results concerning diffusive-Lagrangian aspects of the Navier-Stokes equations. The Euler equations can be written as the active vector system

$$(\partial_t + u \cdot \nabla)\, A = 0$$

where $u = W[A]$ is given by the Weber formula

$$W[A] = \mathbf{P}\left\{(\nabla A)^*\, v\right\}$$

in terms of the gradient of A and the passive field $v = u_0(A)$. ($\mathbf{P}$ is the projector on the divergence-free part.) The initial data is $A(x, 0) = x$, so for short times this is a distortion of the identity map. After a short time one obtains a new u and starts again from the identity map, using the new u instead of u_0 in the Weber formula. The viscous Navier-Stokes equations admit the same representation, with a diffusive back-to-labels map A and a v that is no longer passive. We will discuss how different approximations relate to the exact equations.