Effective boundary condition for creeping flow along a periodic rough surface

B. Cichocki, P. Szymczak

Institute of Theoretical Physics, Warsaw University, ul. Hoża 69, 00-681 Warsaw, Poland,
cichocki@fuw.edu.pl, piotrek@fuw.edu.pl

F. Feuillebois

Physique Thermique, PMMH, ESPCI, 10 rue Vauquelin, 75231 Paris Cedex 05, France
feuillebois@pmmh.espci.fr

Modelling the flow of a viscous fluid along a rough surface is relevant for various applications. A major problem is to ascertain an "equivalent" boundary condition to be applied on a macroscopic scale that is large compared with the roughness scale. The size of the roughness and the fluid velocity on that scale are here assumed to be small enough for the creeping flow equations of fluid motion to apply. The rough surface considered here has periodic corrugations along one dimension. The corrugation profile is symmetric, but its shape is otherwise not restricted. A pure shear flow at infinity is applied along the surface. A flow perpendicular to the corrugations is represented by a biharmonic stream function and a general expression for this function was derived by Hocking [1]. On this basis, the solution is obtained here as an expansion in terms of the slope of corrugations. When the flow at infinity is parallel to the corrugations, the fluid velocity is har! monic and we derive a solution in an analogous way. For both the perpendicular and parallel cases, the rough surface appears to be equivalent to a plane on which the no-slip boundary condition applies. This plane is shifted above the symmetry plane of the corrugations. The expressions for the shift are obtained as series in powers of the slope $s = 2\pi\epsilon/\lambda$, where ϵ is the crest to trough distance and λ is the wavelength of the corrugations. These slowly convergent series then are resummed using Euler's transformation. A further improvement in their convergence is obtained by using the existence of limits of the shift for an infinite slope; the case of a flow parallel to grooves made of semi-infinite parallel planes was solved by Richardson [2] and that of a flow perpendicular to grooves by Hocking [1]. The resulting series for the shift are fast convergent for a wide range of values of the slope s. Earlier results for the flow perpendicular to sinusoidal corrugations [1] are recovered. Results are provided for various shapes of corrugations and grooves, for flows in the perpendicular and parallel directions. By linearity of the Stokes equations, flows at an angle to the corrugations may then be obtained by linear combinations of these two cases.

These theoretical results are presently being used to compare to measurements of the equivalent boundary condition for a sphere settling towards a plane horizontal rough surface [3]. The sphere velocity is measured with laser interferometry, allowing an excellent accuracy.

References

[1] L.M. Hocking. A moving fluid interface on a rough surface. *J. Fluid Mech.*, 76:801–817, 1976.

[2] S. Richardson. A model for the boundary condition of a porous material. Part 2. *J. Fluid Mech.*, 49:327–336, 1971.

[3] N. Lecoq. Étude des interactions hydrodynamiques Particule-parois par interférometrie laser. *PhD Thesis, Université de Rouen, France*, 1994.