Current singularities in two- and three-dimensional MHD

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The development of current singularities and the role they play in mediating fast reconnection is a problem of great interest for laboratory as well as astrophysical plasmas. Unlike hydrodynamics, where vortex singularity formation in two dimensions is forbidden (for flows of finite energy), it is possible, in principle, to form current singularities in two-dimensional magnetohydrodynamics. In a large class of two-dimensional initial conditions, we show that current singularities tend to develop at separatrices algebraically or exponentially in time. In three-dimensional toroidal geometry with no ignorable coordinates, if magnetic surfaces exist, current sheets tend to develop where field lines close on themselves. If the geometry is not toroidal, but has magnetic field nulls, null-null lines are possible sites of singularity formation. There are, however, some three-dimensional geometries that have neither closed magnetic field lines nor magnetic nulls. An example is E. N. Parker's model of the solar corona, which has been a source of controversy in the plasma astrophysics community for the last three decades. Parker proposed that a large-scale magnetic field with a complicated topology does not possess a smooth magnetostatic equilibrium. He considered an ideal plasma column, bounded by perfectly conducting end-plates in which the footpoints of the magnetic fields are frozen. Keeping the footpoints of the magnetic field on one of the fields fixed, the footpoints on the other plate are subjected to slow, random motions that deform the initial magnetic field. Parker claimed that if a sequence of random footpoint motions renders the footpoint mapping sufficiently complicated, there will be no smooth equilibrium for the plasma to relax to. We have recently proved the following theorem: For any given footpoint mapping connected smoothly with the identity mapping, there is at most one smooth equilibrium. It follows as a corollary to this theorem that if such a smooth equilibrium is deformed by further footpoint motion so that it becomes unstable, there is no other smooth equilibrium for the plasma to relax to, and the system must tend to a state containing singular currents. Thus our theorem supports Parker's claim but says nothing about the time required to realize the current singularity. Numerical evidence in support of this theorem will be presented.