Sufficient condition for finite-time singularity in a highly symmetric Euler flow

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We consider the time-evolution of a three-dimensional Euler flow beginning from a highly symmetric initial condition, proposed by S. Kida [1] and simulated by O. Boratav and R. Pelz [2]. Numerical evidence suggests that this highly constrained initial condition produces a singularity in finite time. For this class of initial conditions, it is shown that if the second-order spatial derivative of the pressure is positive following a Lagrangian element (on the x-axis), a finite-time singularity must occur. Under some assumptions, this Lagrangian sufficient condition can be reduced to an Eulerian sufficient condition which requires that the fourth- order spatial derivative of the pressure at the origin be positive for all times leading up to the singularity. Analytical and direct numerical evaluation over a large ensemble of initial conditions demonstrate that for fixed energy, the fourth-order spatial derivative of the pressure is indeed predominantly positive and grows faster than the inverse square of a shrinking length scale. Furthermore, this has the dynamical consequence of reducing the third-order derivative of the velocity at the origin. Assuming that this velocity derivative reaches a minimum value (for a given energy spectrum), we obtain locally self-similar velocity and pressure profiles in reasonable agreement with simulations. The consequences of relaxing some of the Kida symmetries on the formation of the singularity are discussed.

References

[1] Kida, S. (1995). *J. Phys. Soc. Jpn.*, **54**, 2132.

[2] Boratav, O. & Pelz, R. (1994). *Phys. Fluids*, **6**, 2757.