A high-order analog of the helicity number for a pair of divergent-free vector field

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Abstract: Let $B, \tilde{B}$ be a pair of divergent free vector fields in $\Re^3$ with a compact support. We constructs a high-order analog $M(B, \tilde{B})$ of the Gauss helicity number (order 1) $H(B, \tilde{B}) = \int A\tilde{B} d\Re^3$, $curl(A) = B$. The number M is an invariant of volume preserved diffeomorphism with a compact support. It is presented by the following integral expression. For arbitrary 4 points $x_1, x_2, \tilde{x}_1, \tilde{x}_2$ we construct a polylinear function $m(B(x_1), B(x_2), \tilde{B}(\tilde{x}_1), \tilde{B}(\tilde{x}_2))$, invariant under a permutation of the points in each pair. The invariant M is the mean value of m over arbitrary configuration of points.

To clarify the geometrical meaning of the invariant we assume that the fields $B, \tilde{B}$ are concentrated in two disjoin tubes $\{L_1, L_2\}$ with the flows F_1, F_2. We assume that the linking number of the tubes equals zero. Under this assumption we prove

$$M(B) = \beta(L_1, L_2) F_1^2 F_2^2,$$

where β is the Sato-Levine invariant (the Vassiliev invariant of order 3 for isotopy class of 2-component links).