FORMATION AND DISRUPTION OF CONCENTRATED VORTICES IN TURBULENCE

H.K. MOFFATT
Isaac Newton Institute for Mathematical Sciences
20 Clarkson Road
Cambridge CB3 0EH, UK

The Burgers vortex (Burgers 1948) is described by an exact solution of the Navier-Stokes equations in which the effects of uniform axisymmetric stretching are in equilibrium with viscous diffusion. Burgers introduced this vortex as 'a mathematical model illustrating the theory of turbulence', and he noted particularly that the vortex had the property that the rate of viscous dissipation per unit length of vortex was independent of viscosity in the limit of vanishing viscosity (i.e. high Reynolds number). This is of course an attractive feature in the light of Kolmogorov's theory of turbulence, in which the rate of dissipation of energy per unit volume ϵ and the kinematic viscosity ν are regarded as independent variables.

The idea that the small-scale structures of turbulence might be representable in terms of a random distribution of vortex sheets or tubes was taken up by Townsend (1951). Townsend showed that a random distribution of vortex sheets would give rise to an energy spectrum proportional to k^{-2} (multiplied by an exponential viscous cut-off factor) this power-law reflecting the fact that on any straight line through the field of turbulence, intersecting a finite number of vortex sheets, there will be (in the limit of vanishing viscosity) a finite number of discontinuities of velocity per unit length. A random distribution of vortex tubes gave rise to a power-law k^{-1} (again modified by an exponential cut-off), this slower fall-off with k being associated with the more singular behaviour in physical space associated with a line vortex.

The Kolmogorov spectrum $k^{-5/3}$ lies tantalisingly between k^{-1} and k^{-2}, suggesting that the typical (or generic) structures in $\mathbf{x}$-space which may be responsible for such a spectrum should involve some compromise between tubes and sheets, for example spiral structures (Lundgren 1982, Gilbert 1988), these possibly arising through the interaction of tubes and sheets (Krasny 1986, Moffatt 1993).

U. Frisch (ed.), Advances in Turbulence VII, 327–330.

During the last fifteen years, evidence from direct numerical simulation (DNS) of turbulence has accumulated indicating the presence of concentrated tube-like structures in the vorticity field (Siggia 1981, Kerr 1985, Yamamoto & Hosokawa 1988, Vincent & Meneguzzi 1991, and others). This has led to a great revival of interest in the primitive theories of Burgers and Townsend, and a re-evaluation of possible models of turbulence in terms of simple vortex structures.

It was noticed by Kida & Ohkitani (1992) that the dissipation structure in the concentrated vortices of 3D turbulence exhibit two maxima, off-set from the centre of the vortex, and they suggested that this might be explained in terms of the action of non-axisymmetric strain acting on the vortex. This suggestion was taken up by Moffatt, Kida & Ohkitani (1994) who developed a high Reynolds number asymptotic theory of a vortex subjected to non-axisymmetric strain. Determination of the dissipation structure involved pursuing the analysis to third order in the small parameter Re^{-1}, and this analysis did indeed reveal the two peaks in the dissipation structure, arising from a symmetry-breaking splitting of the circle of maximum dissipation that occurs for the Burgers vortex. Remarkably, at leading order in the asymptotic analysis, the axisymmetric Burgers vortex emerges in spite of the non-axisymmetric character of the strain. This is because, at high vortex Reynolds number, the vortex spins rapidly in the strain field, and experiences the θ-averaged strain, which is axisymmetric. A similar behaviour had been previously recognised by Ting & Tung (1965), and by Neu (1984). Even more remarkably, the solution of Moffatt, Kida & Ohkitani indicates that the vortex can survive for an exponentially long time even when one of the rates of strain in the plane of cross-section of the vortex is positive. Again, this is because it is only the θ-averaged strain that is relevant at leading order.

The characteristic dissipation structures identified by Moffatt, Kida & Ohkitani are present also, although for rather different reasons, in two-dimensional freely decaying turbulence, at the stage when identifiable vortices emerge from a random initial state (McWilliams 1984, 1990, Jiménez, Moffatt & Vasco 1996). Each vortex in such a field moves with the local velocity induced by all the other vortices, and is also subject to the two-dimensional strain field associated with the presence of all the other vortices. At high Reynolds number, the effect of this strain field is to distort each vortex cross-section to slightly elliptical form; the associated dissipation field has precisely the same structure as that determined in the earlier work of Moffatt, Kida & Ohkitani. This remarkable result is a consequence of the analogy between steady stretched three-dimensional vortices and unsteady unstretched two-dimensional vortices, as described by Lundgren (1982).

The elliptic deformation of vortices in both two- and three-dimensional turbulence makes them prone to the type of three-dimensional resonant instability identified by Bayly (1986) and Pierrehumbert (1986). As shown, however, by Le Dizès, Rossi & Moffatt (1996), stretching carries the wave-number of sinusoidal perturbations through the unstable wave-number band in a finite time, so that infinitesimal disturbances are always asymptotically stable. This mechanism is not present for two-dimensional (unstretched) vortices, and one may reasonably conjecture that 2D turbulence is always unstable to 3D disturbances (in the absence of stabilising mechanisms such as stratification or magnetic field).

A serious limitation of the Burgers model in the context of three-dimensional turbulence lies in the assumption of the uniformity of the strain field, and the associated infinite length of the stretched vortices. In fact, the region of concentration is of finite length, the vortex lines diverging more or less rapidly at the ends of these regions of concentration. This finite length is apparent also in experiments (Douady, Couder & Brachet 1991) designed to detect intense vortex filaments in turbulent flow of liquids seeded with small gas bubbles.

Variation of the strain field arises through the non-uniform action of the 'other vortices' near to the parent vortex whose structure is considered. In so far as these other vortices may be treated as point vortices, the non-uniform strain field acting on the parent vortex is a strain field associated with a non-uniform potential flow. In this lecture, a simple model will be developed involving the action of distributed vortices on a two-dimensional stretched vortex sheet (of Burgers type). The problem is treated by using the potential ϕ and stream function ψ of the non-uniform straining flow as independent coordinates. The advection-diffusion equation for the parent vortex sheet has universal form in terms of these coordinates, and a wide family of exact solutions of the Navier-Stokes equations is thus generated. A variety of solutions will be described, which provide a good indication of the manner in which such vortex sheets may disrupt in regions of strong non-uniformity of strain.

A similar technique runs into difficulties for the analogous axisymmetric problem, because in this case the advection-diffusion equation, expressed in terms of the relevant ϕ and ψ, is not universal. Nevertheless, the approach does indicate one mechanism by which vortex disruption can occur.

References

Bayly, B. (1986) Three-dimensional instability of elliptical flow, *Phys. Rev. Lett.* **57**, p. 2160.

Burgers, J.M. (1948) A mathematical model illustrating the theory of turbulence, *Adv. Appl. Mech.* **1**, pp. 171–199.

Douady, S., Couder, Y. & Brachet, M.E. (1991) Direct observation of the intermittency of intense vorticity filaments in turbulence, *Phys. Rev. Lett.* **67**, 983–986.

Gilbert, A.D. (1988) Spiral structures and spectra in two-dimensional turbulence, *J. Fluid Mech.* **193**, pp. 475–497.

Jiménez, J., Moffatt, H.K. and Vasco, C. (1996) The structure of the vortices in freely decaying two-dimensional turbulence, *J. Fluid Mech.* **313**, pp. 209–222.

Kerr, R.M. (1985) Higher-order derivative correlations and the alignment of small-scale structures in isotropic numerical turbulence, *J. Fluid Mech.* **153**, pp. 31–58.

Kida, S. and Ohkitani, K. (1992) Spatio-temporal intermittency and instability of a forced turbulence, *Phys. Fluids* A4, pp. 1018–1027.

Krasny, R. (1986) Desingularisation of periodic vortex sheet roll-up, *J. Computational Physics* **65**, pp. 292–313.

Le Dizès, S., Rossi, M. & Moffatt, H.K. (1996) On the three-dimensional instability of elliptical vortex subjected to stretching, *Phys. Fluids* **8**(8), pp. 2084–2090.

Lundgren, T. (1982) Strained spiral vortex model for turbulent fine structure, *Phys. Fluids* **25**, pp. 2193–2203.

McWilliams, J.C. (1984) The emergence of isolated coherent vortices in turbulent flow, *J. Fluid Mech.* **146**, pp. 21–43.

McWilliams, J.C. (1990) The vortices of two-dimensional turbulence, *J. Fluid Mech.* **219**, pp. 361–385.

Moffatt, H.K. (1993) Spiral structures in turbulent flow, *New Approaches and Concepts in Turbulence, Monte Verità*, Birkhäuser Verlag Basel, pp. 121–129.

Moffatt, H.K., Kida, S. and Ohkitani, K. (1994) Stretched vortices – the sinews of turbulence; large-Reynolds-number asymptotics, *J. Fluid Mech.* **259**, pp. 241–264.

Neu, J.C. (1984) The dynamics of stretched vortices, *J. Fluid Mech.* **143**, pp. 253–276.

Pierrehumbert, R.T. (1986) Universal short-wave instability of two-dimensional eddies in an inviscid fluid, *Phys. Rev. Lett.* **57**, p. 2157.

Siggia, E.D. (1981) Numerical study of small scale intermittency in three-dimensional turbulence, *J. Fluid Mech.* **107**, pp. 375–406.

Ting, L. and Tung, C. (1965) Motion and decay of a vortex in a nonuniform stream, *Phys. Fluids* **8**, pp. 1039–1051.

Townsend, A.A. (1951) On the fine scale structure of turbulence, *Proc. R. Soc. Lond.* A **208**, pp. 534–542.

Vincent, A. and Meneguzzi, M. (1991) The spatial structure and statistical properties of homogeneous turbulence, *J. Fluid Mech.* **225**, pp. 1–25.

Yamamoto, K. and Hosokawa, I. (1988) A decaying isotropic turbulence pursued by the spectral method, *J. Phys. Soc. Japan* **57**, pp. 1532–1535.