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What maintains the Earth's magnetic field? Why does the solar magnetic field follow a 22-year cycle? How can we explain the significant magnetic fields on planets like Jupiter and Saturn? Dynamo theory is starting to answer these questions

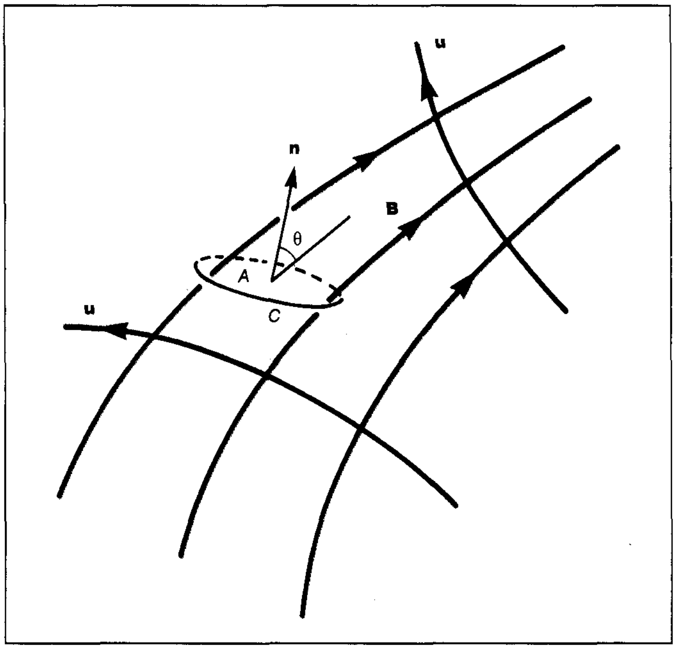
# Cosmic dynamos: from alpha to omega

**KEITH MOFFATT**

ENERGY manifests itself in many forms in the cosmos, chief among these being kinetic, thermal, gravitational, nuclear and magnetic. Magnetic energy is associated with electric currents flowing in conducting matter, such as the ionised gas of the interstellar medium, the insides of stars, or the deep liquid metal interiors of planets like the Earth. In the absence of electromotive forces (like those in the humble chemical battery), these electric currents, and the magnetic fields they generate, will always decay and the magnetic energy will in turn be converted to heat.

A fundamental challenge in physics is to explain why, despite this irreversible and unremitting dissipation of magnetic energy, magnetic fields are such a universal phenomenon. We need a mechanism that can systematically convert the kinetic energy of fluid motion into magnetic energy and thus compensate for the persistent erosion of currents, and their associated magnetic fields, due to the non-zero resistivity of the fluid. Such a mechanism is known as a dynamo.

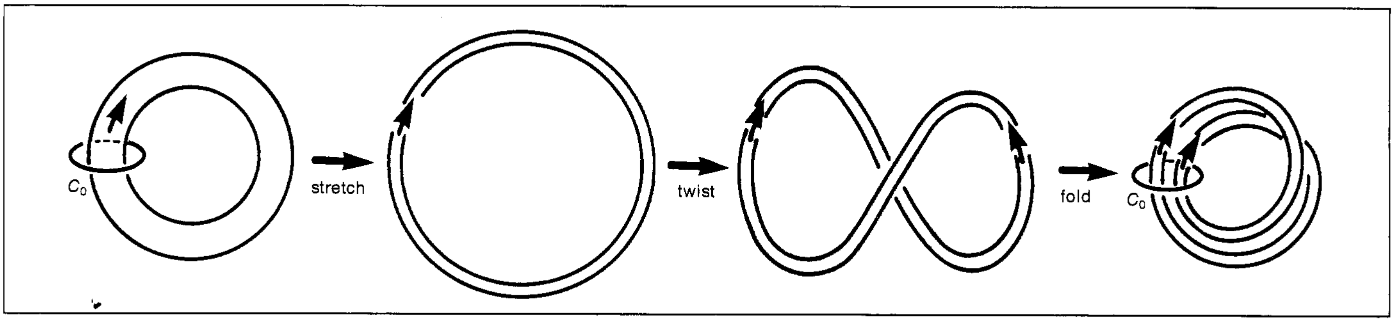
The problem appears most acute for the Earth, whose magnetic field is a phenomenon of wonder and mystery for any child who has ever played with a magnetic compass. We know that the dominant source of the "geomagnetic field" is a system of electric currents in the deep interior of the Earth. We know also that, in the absence of any regenerative mechanism, these currents would vanish in a time of the order of $10^4$ years. (The decay time, $T$, for currents in a sphere of radius, $r$, and resistivity, $\eta$, is $T \sim r^2/\pi^2\eta$. The Earth's metallic core has $r \approx 3500$ km and $\eta \approx 3$ m$^2$ s$^{-1}$, therefore $T$ is about $4 \times 10^{11}$ s or 13 000 years.) So why do palaeomagnetic records indicate that the geomagnetic field has existed at approximately its present strength ($\sim 0.3$ Gauss) for at least $10^8$ years, a thousand times longer than any primitive theory of solid conductors would indicate? Why does the dominant ingredient of the field, the dipole, oscillate with periods $\sim 10^3$–$10^4$ years? And why does the North–South polarity of the geomagnetic field randomly reverse at intervals $\sim 10^6$ years?



**1** Magnetic lines of force, "**B**-lines", are convected by the fluid flow, **u**, as if they were elastic strings embedded in the fluid. As the "strings" are stretched, the magnetic field becomes stronger. The magnetic flux through a small closed loop, $C$, moving with the flow is $\Phi = AB\cos\theta$. Flux is conserved if the effects of resistivity are negligible

Satellite observations and magnetographs reveal that the Sun also has a complex and ever-changing magnetic field, and that this is strongly correlated with the granulation pattern of upwellings and downwellings on the solar surface. The natural decay time for the solar magnetic field is much longer than for the geomagnetic field (the Sun has a much larger radius and a smaller resistivity). In fact this decay time is of the order of the age of the Sun ($\sim 4 \times 10^9$ years). There is not, therefore, the same pressing need for a dynamo theory of the solar magnetic field – it could conceivably be a relic of a magnetic field trapped in the Sun during its formation. However, the main solar field exhibits periodic behaviour with the same 22-year period as the sunspot cycle; it is difficult to comprehend such short-term variability other than through a dynamo mechanism.

One of the most remarkable features of dynamo theory is that, despite the extreme disparity of physical conditions in the Earth and Sun, the same basic principles are potentially capable of explaining the observed magnetic



**2** Stretch–twist–fold cycle whereby the magnetic flux through a *fixed* loop, $C_0$, in a conducting fluid may be doubled like the tension in an elastic band

behaviour of both. It is a small step to extend the theory to those planets with significant magnetic fields (Jupiter, Saturn, Uranus and Neptune); and to the countless stars, like the Sun, whose magnetic fields can now be detected by the Zeeman splitting of their spectral lines.

## Frozen fields

Since the time of Faraday, it has been illuminating to think of magnetic fields in terms of magnetic lines of force that behave like elastic strings threading their way through the conducting medium. This picture is particularly helpful in the electrodynamics of conducting fluids, known as magnetohydrodynamics (MHD – see box); one of the fundamental theorems of MHD is that in a perfectly conducting (i.e. zero resistivity) fluid, particles that lie on a line of force at any instant continue to lie on a line of force for all subsequent times. This allows us to identify magnetic lines of force with material lines moving with the fluid: we say that the magnetic field is "frozen" in the fluid. The velocity field, **u**, of the fluid simply carries the magnetic field lines, **B**, as if they were elastic strings (figure 1). Figure 1 also shows a small closed material curve, *C*, bounding an area, *A*, through which there is a non-zero magnetic flux, $\Phi \sim AB\cos\theta$, where *B* is the magnitude of **B** and $\theta$ is the angle between **B** and **n**, the vector normal to *C*. As *C* moves with the fluid, this flux is trapped and remains constant. If the projected area ($A\cos\theta$) decreases, then *B* must increase to keep the flux ($\Phi$) constant. Therefore, for incompressible flow, stretching the lines of force implies a proportionate increase in *B* (just as it would increase the tension in elastic strings).

## MHD made easy

Magnetohydrodynamics (MHD) is the study of the interaction of the magnetic field, **B**, and flow, **u**, of a conducting fluid. Ampère's law tells us that there is a current density associated with the magnetic field ($\mathbf{j} = \mathrm{curl}\mathbf{B}/\mu_0$ where $\mu_0$ is permeability). This current leads to a Lorentz force ($\mathbf{j} \times \mathbf{B}$) on the fluid which is perpendicular to both the magnetic field and the current.

Faraday's law tells us that changes in magnetic field strength with time give rise to an electric field ($\partial\mathbf{B}/\partial t = -\mathrm{curl}\mathbf{E}$); the interaction of **u** and **B** also gives rise to an electric field ($\mathbf{u} \times \mathbf{B}$). Ohm's law tells us that the current generated by these fields is $\mathbf{j} = \sigma(\mathbf{E}+\mathbf{u} \times \mathbf{B})$ where $\sigma$ is conductivity.

These relations can be combined to give one equation governing the time evolution of the magnetic field

$$\partial\mathbf{B}/\partial t = \mathrm{curl}(\mathbf{u} \times \mathbf{B}) + \eta\nabla^2\mathbf{B}$$

where $\eta = (\mu_0\sigma)^{-1}$ is the resistivity.

The time evolution of the fluid is determined by the Navier–Stokes equation

$$\partial\mathbf{u}/\partial t + \mathbf{u}.\mathrm{grad}\mathbf{u} = -(1/\rho)\mathrm{grad}p + \mathbf{j} \times \mathbf{B} + \nu\nabla^2\mathbf{u} + \Theta\mathbf{g}$$

where $\rho$ is the density, $p$ the pressure, $\nu$ the kinematic viscosity, $\Theta$ the buoyancy field and **g** gravitational acceleration.

MHD is essentially the solution of these coupled time-dependent partial differential equations in three dimensions for real fluids with non-zero resistivity. Combining electromagnetism and fluid mechanics in this way also leads to new physical effects, such as Alfvén waves, which are not present in either individually.

One common approximation in MHD, due to Alfvén, is that the fluid is perfectly conducting and that the magnetic flux lines are "frozen" into the fluid (figure 1). However, this is only valid for timescales much shorter than $\tau = L^2/\eta$ where *L* is the length-scale of the problem.

We can now see the seeds of dynamo action: if we can devise a flow which stretches the lines of force without limit, then the field intensity will grow from an arbitrarily weak level to a level that is not only detectable, but also capable of reacting back dynamically (through the Lorentz force) on the flow responsible for this, otherwise unlimited, growth. Here again, the elastic string analogy is helpful. Just as we may increase the tension in an elastic band by stretching, twisting and folding it, so the intensity of a magnetic field confined to a single flux tube can be doubled by the same "stretch–twist–fold" sequence (figure 2). But there is a price to pay if the stretch–twist–fold sequence is repeated many times (which is possible, in principle, with a magnetic field but physically impossible with an elastic band). An increasingly complex structure builds up on the scale of the flux tube cross-section and at some stage the resistivity of the fluid, no matter how small it may be, will lead to Joule dissipation (the production of heat from magnetic energy at a rate proportional to the resistivity times the current squared).

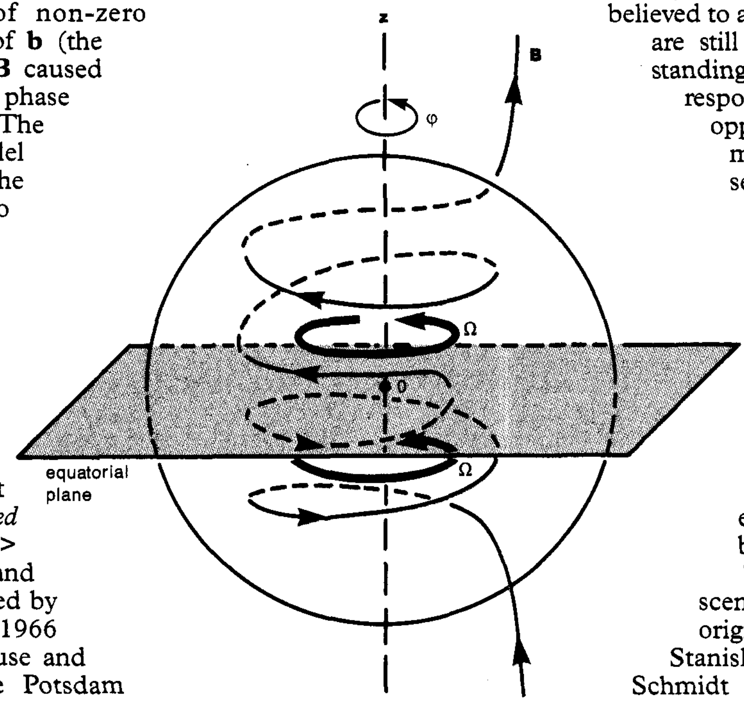
The large magnetic field gradients in this example imply large currents and hence severe Joule dissipation. The first problem of dynamo theory is to establish that the average value of the magnetic field can continue to grow despite the increasing influence of Joule dissipation. The stretch–twist–fold mechanism, devised originally in 1972 by the late Jacov Borisovich Zel'dovich of the Institute of Cosmic Research, Moscow, is only illustrative. However, two other, more realistic, ways for increasing the magnetic field intensity do exist – differential rotation and the $\alpha$-effect.

## Alpha and omega

Imagine a rotating sphere of fluid in which the rate of rotation, $\Omega$, about the axis of the sphere, *Oz*, increases as the axis is approached (figure 3). Such a state of "differential rotation" tends to be established when convection currents, driven by buoyancy forces, are also influenced by Coriolis forces: conservation of angular momentum makes the descending fluid elements rotate more rapidly (the "ballerina effect"). In so far as the magnetic field lines threading through the fluid are "frozen in", they move with the fluid and are "cranked" round the axis $Oz$. This leads to a large azimuthal component of field, $B_\varphi$. Note that this component is antisymmetric about the "equatorial" plane perpendicular to $Oz$.

The other mechanism is more subtle and requires us to go beyond the frozen-field principle. The "α-effect" allows a straight line of force, along the $x$-axis say, to be converted into a helix by a velocity field which is circularly polarised (in the $yz$ plane in this case – see figure 4). The simplest velocity field, $\mathbf{u}$, of this form has the components 0, $u_0\cos(kx-\omega t)$, $u_0\sin(kx-\omega t)$, representing a circularly polarised wave propagating in the $x$-direction with velocity $c = \omega/k$. This velocity field has the remarkable property that its vorticity, $\omega \equiv \mathrm{curl}\,\mathbf{u}$, is parallel to itself (it can be easily verified that $\omega = -k\mathbf{u}$). It follows that the helicity of this flow, $\mathbf{u}.\omega = -k\mathbf{u}^2 = -k\mathbf{u}_0^2$, is uniform in space. Helicity, unlike vorticity, distinguishes "right-handedness" from "left-handedness" in fluid motion. It is this helicity that is responsible for creating the helical lines of force.

Now a line current flowing parallel to the **B**-line would likewise have the effect of replacing the straight **B**-line by a helical **B**-line (the line current produces a circular magnetic field around the conductor and the addition of this field to the straight **B**-field produces the helical field). It is tempting to interpret the creation of this helix in terms of an induced current due to the helicity of the flow field. This conclusion is in fact correct, although the argument is simplistic; the current only appears if the effects of non-zero resistivity shift the phase of $\mathbf{b}$ (the magnetic perturbation to $\mathbf{B}$ caused by the flow) relative to the phase of the velocity field, $\mathbf{u}$. The resulting current, $\mathbf{j}$, is parallel to the initial field, $\mathbf{j} = \alpha\mathbf{B}$; the constant $\alpha$ is proportional to the phase shift, and hence to the fluid resistivity (in figure 4 $\alpha$ is positive).



3 Differential rotation: if the rotation rate, Ω, of the fluid increases with depth, then a magnetic field line passing through the sphere will tend to be cranked around the axis, $Oz$. This generates an azimuthal component of magnetic field which can be quite large

At first it seems surprising that $\mathbf{j}$ and $\mathbf{B}$ are parallel because the electromotive force $\mathbf{u} \times \mathbf{B}$ associated with Faraday's law of induction is *perpendicular* to $\mathbf{B}$; however, when $\mathbf{u}$ and $\mathbf{b}$ are not in phase, the *space-averaged* electromotive force $<\mathbf{u} \times \mathbf{b}>$ is indeed parallel to $\mathbf{B}$, and this is the "α-effect" revealed by this simple example. In 1966 Max Steenbeck, Fritz Krause and Karl-Heinz Rädler of the Potsdam Institut für Astrophysik in Germany showed that the same effect occurs in any turbulent flow field provided the mean helicity, $<\mathbf{u}.\omega>$, is non-zero. However, the actual evaluation of $\alpha$ presents a number of major difficulties.

We can now combine these two fundamental mechanisms to see how a magnetic field may be generated in a spherical fluid conductor, by what is known as the αΩ ("alpha–omega") mechanism. The total field can be decomposed into a poloidal ("North–South") ingredient, $\mathbf{B}_P$, whose lines of force lie in meridian planes, and a toroidal ("East–West") ingredient, $\mathbf{B}_T$, whose lines of force are circles about the rotation axis $Oz$. We have seen that differential rotation generates $\mathbf{B}_T$ from $\mathbf{B}_P$. Likewise, the α-effect, which converts each $\mathbf{B}_T$-line to a helix wrapped on a torus around $Oz$, generates $\mathbf{B}_P$ from $\mathbf{B}_T$. Resistive diffusion allows this newly generated $\mathbf{B}_P$ to reinforce the field $\mathbf{B}_P$ that we started with.

We thus have a regenerative cycle:

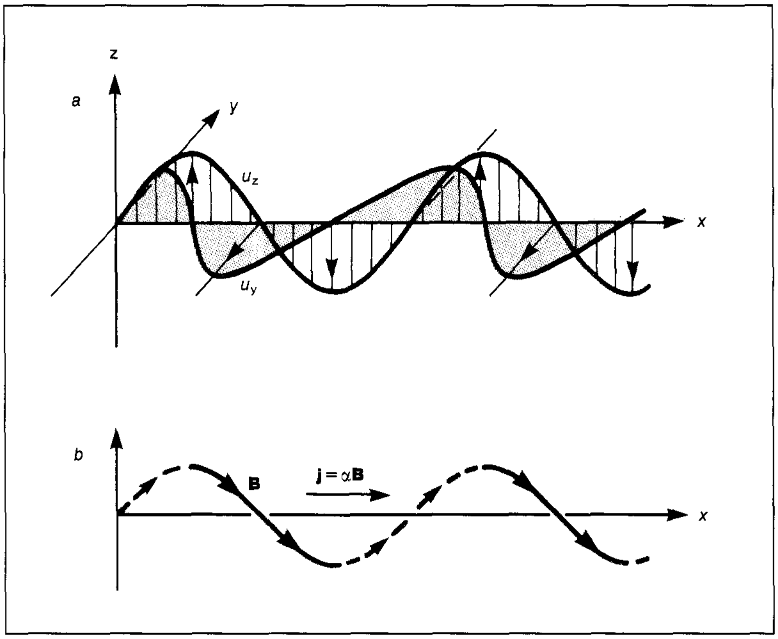
$$\begin{tikzcd} \mathbf{B}_P \arrow[r, bend left, "\text{differential rotation } (\Omega)"] & \mathbf{B}_T \arrow[l, bend left, "\alpha\text{-effect plus diffusion}"] \end{tikzcd}$$

This cycle is believed to be at the heart of both the geodynamo and the solar dynamo. It should be noted that without some resistivity (and its associated diffusion of field), this regenerative cycle cannot operate. Thus the very mechanism (Joule dissipation) that causes the decay of magnetic fields is also necessary for their regeneration.

## Outstanding problems

The Earth and the Sun are depicted schematically in figure 5. In both, the convective region is a spherical annulus, the liquid outer core for the Earth, and the convection zone in the Sun. There the similarity apparently ends, for the convective velocities are $\sim$mm s$^{-1}$ in the Earth's core, $\sim$km s$^{-1}$ in the solar convection zone. And yet the same type of αΩ dynamo mechanism is believed to apply to both! However, there are still major gaps in our understanding of the *dynamics* of the flows responsible for dynamo action (as opposed to the purely electromagnetic, or kinematic, consequences) in both the Earth and the Sun. Some of the outstanding problems in the field were tackled last year by about 30 international geophysicists, astrophysicists and fluid dynamicists who participated in the six-month Dynamo Theory Programme at the new Isaac Newton Institute for Mathematical Sciences in Cambridge, UK.

The most widely accepted scenario for the Earth is that originally formulated in 1963 by Stanislav Braginsky, then at the Schmidt Institute of Physics of the Earth in Moscow – Braginsky suggested that slow solidification of the liquid at the inner core boundary (ICB) releases a slight excess of lighter ingredients (sulphur for example) in the alloy which makes up the liquid outer core. The resulting buoyancy generates convection currents as the light fluid rises in blobs or plumes towards the core–mantle boundary (CMB), mixing with the surrounding fluid in the process. As yet, however, there is merely speculation concerning the length-scale of the rising elements, the degree of turbulence that they generate, and the rate at which they mix with the heavier material in the liquid outer core (the turbulent diffusion rate). At one extreme, the convection currents may be large eddies on the scale (approximately 2000 km) of the depth of the convection zone; this is the view advanced by Braginsky, now at the University of California at Los Angeles, within the framework of his controversial "Model-*Z*" in which the magnetic lines of force in the core are nearly aligned with the axis of rotation (the direction *Oz*).



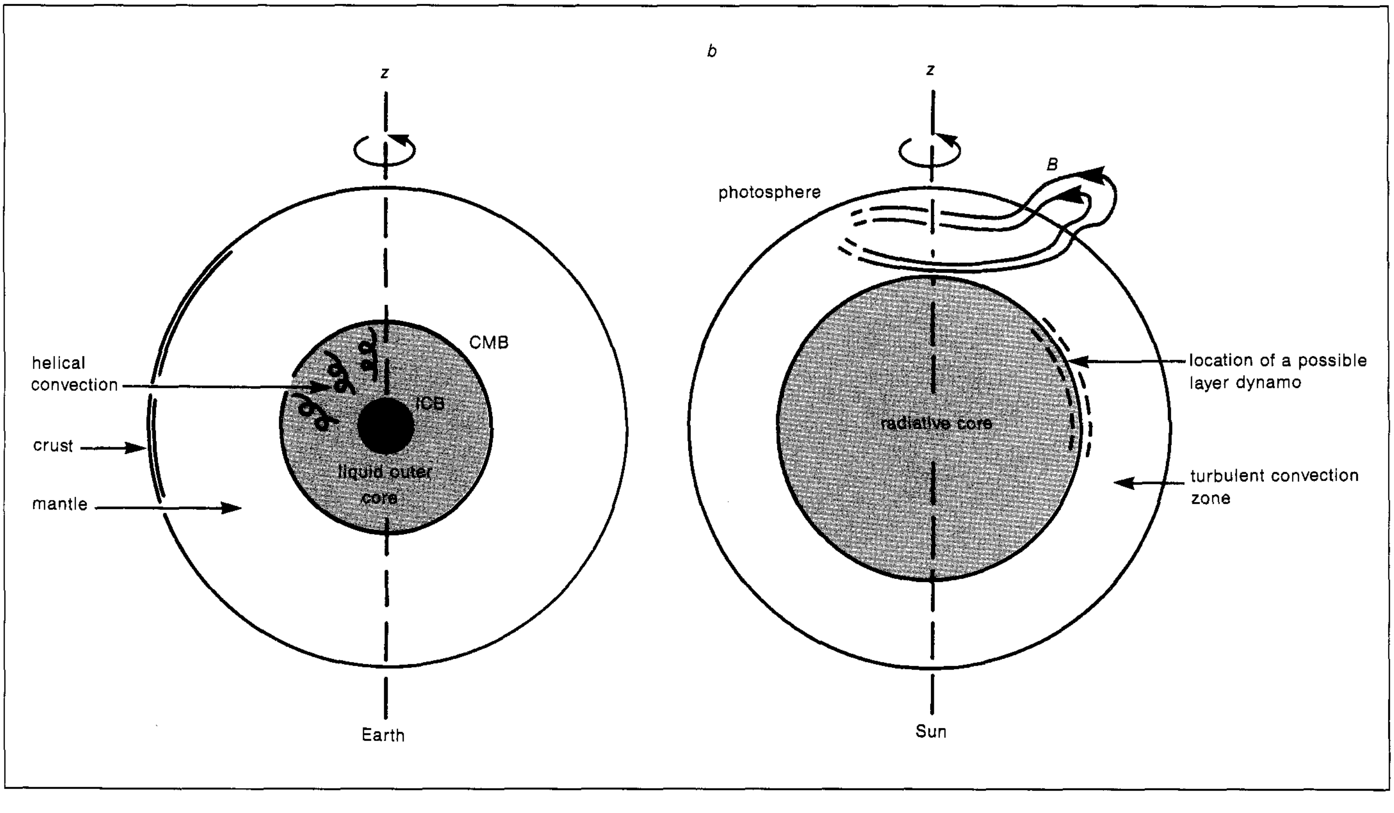
**4** (*a*) A velocity field with components $u_x = 0$, $u_y = u_0\cos(kx-\omega t)$, $u_z = u_0\sin(kx-\omega t)$. This field is circularly polarised in the *yz* plane and can deform a magnetic line of force along the *x*-axis into a helix (*b*)

At the other extreme, blobs on the scale of 10–100 km may retain their integrity, like rising bubbles, throughout their journey from the ICB to the CMB. In this scenario the blobs follow a helical path on a paraboloid of revolution about the rotation axis. Detailed study of the variation of the Earth's field with time, and resulting reconstruction of the field (by inverse theory) at the CMB, is potentially capable of distinguishing which of these pictures (if either!) is nearer to the truth. In the meantime, theorists are striving to develop models that are at least internally consistent, a task that is still at the absolute limits of the most powerful supercomputers.

The problems are no less acute for the Sun, but they have a rather different character. One problem relates to the phenomenon of "magnetic buoyancy": the pressure in a fluid is the sum of the magnetic pressure (which is proportional to the square of the magnetic field strength) and the fluid pressure. The fluid pressure and density therefore tend to be low where the magnetic field is strong. This means that magnetic flux tubes generated in the solar convection zone tend to rise and erupt through the surface. This would be fine if the timescale of rise were of the order of the 22-year sunspot cycle period; but the simplest theory of magnetic buoyancy, first developed in 1955 by Eugene Parker of the University of Chicago, suggests a much shorter timescale of the order of months or less – so short that a dynamo process located in the convection zone appears to be quite at variance with observed solar activity.

Attempts are currently being made to circumvent this

**5** (*a*) The geodynamo process is located in the liquid outer core of the Earth. Helical convection is driven by a process of gravitational segregation and there is an associated slow growth of the solid inner core. (*b*) The solar dynamo is driven by turbulence and differential rotation in the convection zone or, possibly, by instabilities confined to a layer at the bottom of the convection zone. Magnetic flux erupts periodically through the surface (photosphere) giving the well-known 22-year cycle of magnetic activity



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difficulty: the solar rotation (with a period of about 27 days) undoubtedly impedes the eruption process, but perhaps not sufficiently; alternatively, the true seat of dynamo action may be in a layer near the bottom of the convection zone where a stable density gradient may more effectively impede the rise of magnetic flux tubes – much as if they were imbedded in a layer of treacle!

The new science of solar seismology (see "Deep secrets of the Sun" *Physics World* March 1991 p40), which uses the increasingly detailed observations of oscillations and waves on the solar surface to infer the density, rotation and magnetic field in the interior, has the potential to resolve this issue, but the clues obtained so far do not yet fit easily into any pattern consistent with the alluring $\alpha\Omega$-dynamo mechanism.

## Turbulence ahead

A second problem which attracted heated discussion at the Isaac Newton Institute, a discussion that Newton himself would have relished, related to the role of turbulent diffusion associated with random motion of fluid elements in the solar dynamo. Molecular diffusion is far too weak to catalyse the dynamo process on the required 22-year time-scale, making turbulent diffusion an essential ingredient. However, a dynamo-generated magnetic field suppresses the very turbulence on which it feeds; does this suppression effect kill the dynamo, or does it merely nudge it into another mode of operation (e.g. one in which regions of strong turbulence and strong magnetic field are separate but still able to interact across their common frontier)?

The Earth and the Sun are the two most familiar examples of cosmic bodies containing self-generated magnetic fields. But the generality of the dynamo mechanism suggests that any self-gravitating rotating mass of conducting fluid – whether planet, star or galaxy – will exhibit a magnetic field generated by the $\alpha\Omega$, or similar, mechanism. The general theory is robust, but reconciliation of detailed theory and observation in each individual case is certain to present challenges of exceptional difficulty.

## Further reading

S I Braginsky 1991 Towards a realistic theory of the geodynamo *Geophys. Astrophys. Fluid Dyn.* **60** 89–134

D Gough and J Toomre 1991 Seismic observations of the solar interior *Ann. Rev. Astron. Astrophys.* **29** 627–684

H K Moffatt and A Tsinober 1992 Helicity in laminar and turbulent flow *Ann. Rev. Fluid Mech.* **24** 281–312

P H Roberts and A M Soward 1992 Dynamo theory *Ann. Rev. Fluid Mech.* **24** 459–512

R Rosner and N O Weiss 1992 The origins of the solar cycle in *The Solar Cycle* K L Harvey ed (Astronomy Society of the Pacific, San Francisco) **27** 511–531

The following books arising from the Dynamo Theory Programme of the Isaac Newton Institute in Cambridge will be published soon:

M R E Proctor and A D Gilbert eds 1993 *Lectures on Solar and Planetary Dynamos* (Cambridge University Press, Cambridge)

M R E Proctor, P Matthews and A Rucklidge eds 1993 *Theory of Solar and Planetary Dynamos* (Cambridge University Press, Cambridge)

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